JEE Main Maths Trigonometry Previous Year Questions With Solutions

Question 1: The general solution of sin x – 3 $\sin^2 x$ + $\sin^3 x$ = cos x – 3 $\cos^2 x$ + $\cos^3 x$ is _________.

Solution:

$\sin x - 3\sin^2 x + \sin^3 x = \cos x - 3\cos^2 x + \cos^3 x$

$\Rightarrow 2\sin^2 x \cos x - 3\sin^2 x - 2\cos^2 x \cos x + 3\cos^2 x = 0$

$\Rightarrow \sin^2 x\,(2\cos x - 3) - \cos^2 x\,(2\cos x - 3) = 0$

$\Rightarrow (\sin^2 x - \cos^2 x)(2\cos x - 3) = 0$

$\Rightarrow \sin^2 x = \cos^2 x$

$\Rightarrow 2x = 2n\pi \pm (\pi / 2 - 2x)$ i.e.,

$x = n\pi / 2 + \pi / 8$

Question 2: If sec 4θ – sec 2θ = 2, then the general value of θ is __________.

Solution:

$\sec 4\theta - \sec 2\theta = 2 \Rightarrow \cos 2\theta - \cos 4\theta = 2\cos 4\theta \cos 2\theta$

$\Rightarrow -\cos 4\theta = \cos 6\theta$

$\Rightarrow 2\cos 5\theta \cos\theta = 0$

$\Rightarrow H = [h \cot 15^o] / [\cot 15^o - 1]$ or

$n\pi/5 + \pi/10$

Question 3: If tan (cot x) = cot (tan x), then sin 2x = ___________.

Solution:

$\tan(\cot x) = \cot(\tan x) \Rightarrow \tan(\cot x) = \tan(\pi / 2 - \tan x)$

$\cot x = n\pi + \pi / 2 - \tan x$

$\Rightarrow \cot x + \tan x = n\pi + \pi / 2$

$2 \sin 2x = n\pi + \pi / 2$

$\Rightarrow \sin 2x = 2 / [n\pi + \{\pi / 2\}]$

$= 4 / \{(2n + 1)\pi\}$

Question 4: If the solution for θ of cospθ + cosqθ = 0, p > 0, q > 0 are in A.P., then numerically the smallest common difference of A.P. is ___________.

Solution:

Given $\cos p\theta = -\cos q\theta = \cos(\pi + q\theta)$

$p\theta = 2n\pi \pm (\pi + q\theta), n \in I$

$\theta = [(2n + 1)\pi] / [p - q]$ or $[(2n - 1)\pi] / [p + q], n \in I$

Both the solutions form an A.P. $\theta = [(2n + 1)\pi] / [p - q]$ gives us an A.P. with common difference $2\pi / [p - q]$ and $\theta = [(2n - 1)\pi] / [p + q]$ gives us an A.P. with common difference = $2\pi / [p + q]$.

Certainly, $\{2\pi / [p + q]\} < \{|2\pi / [p - q]|\}$.

Question 5: If a, b are different values of x satisfying a cosx + b sinx = c, then tan ([α + β] / 2) = ______________.

Solution:

a cosx + b sinx = c ⇒ a $\{[(1 - \tan^2 (x / 2)] / [1 + \tan^2 (x / 2)]\}$ + 2b $\{[\tan (x / 2) / 1 + \tan^2 (x / 2)\}$ = c

⇒ $(a + c) * \tan^2 [x / 2] - 2b \tan [x / 2] + (c - a) = 0$

This equation has roots tan [α / 2] and tan [β / 2].

Therefore, tan [α / 2] + tan [β / 2] = 2b / [a + c] and tan [α / 2] * tan [β / 2] = [c – a] / [a + c] Now

tan (α / 2 + β / 2) = {tan [α / 2] + tan [β / 2]} / {1 – tan [α / 2] * tan [β / 2]} = {[2b] / [a + c]} / {1– ([c – a] / [a + c])} = ba

Question 6: In a triangle, the length of the two larger sides are 10 cm and 9 cm, respectively. If the angles of the triangle are in arithmetic progression, then the length of the third side in cm can be _________.

Solution:

We know that in a triangle larger the side, larger the angle.

Since angles ∠A, ∠B and ∠C are in AP.

Hence, ∠B = 60^o cosB = $[a^2 + c^2 - b^2] / [2ac]$

⇒ $1 / 2 = [100 + a^2 - 81] / [20a]$

⇒ $a^2 + 19 = 10a$

⇒ $a^2 - 10a + 19 = 0$

$a = 10 \pm (\sqrt{[100 - 76]} / [2])$

⇒ $a + c\sqrt{2} = 5 \pm \sqrt{6}$

Question 7: In triangle ABC, if ∠A = 45◦, ∠B = 75◦, then $a + c\sqrt{2}$ = __________.

Solution:

$\angle C = 180^o - 45^o - 75^o = 60^o$

Therefore, $a + c\sqrt{2} = k (\sin A + \sqrt{2} \sin C)$

$= k (1 / \sqrt{2} + [\sqrt{3} / 2] * \sqrt{2})$

$= k ([1 + \sqrt{3}] / [\sqrt{2}])$ and

$k = b / [\sin B]$

$\Rightarrow a + c\sqrt{2} = b / \sin 75^\circ ([1 + \sqrt{3}] / [\sqrt{2}])$

$= 2b$

Question 8: If $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$ then $p^2 + q^2 + r^2 + 2pqr$ = ________.

Solution:

According to given condition, we put $p = q = r = 1 / 2$

Then, $p^2 + q^2 + r^2 + 2pqr = (1 / 2)^2 + (1 / 2)^2 + (1 / 2)^2 + 2 * [1 / 2] * [1 / 2] * [1 / 2]$

$= [1 / 4] + [1 / 4] + [1 / 4] + [2 / 8]$

$=1$

Question 9: $\tan [(\pi / 4) + (1 / 2) * \cos^{-1} (a / b)] + \tan [(\pi / 4) - (1 / 2) \cos^{-1} \cdot(a / b)]$ = __________.

Solution:

$\tan [(\pi / 4) + (1 / 2) * \cos^{-1} (a / b)] + \tan [(\pi / 4) - (1 / 2) \cos^{-1} \cdot(a / b)]$

Let $(1 / 2) * \cos^{-1} (a / b) = \theta$

$\Rightarrow \cos 2\theta = a / b$

Thus, $\tan [\{\pi / 4\} + \theta] + \tan [\{\pi / 4\} - \theta] = [(1 + \tan\theta) / (1 - \tan\theta)] + ([1 - \tan\theta] / [1 + \tan\theta])$

$= [(1 + \tan\theta)^2 + (1 - \tan\theta)^2] / [(1 - \tan^2\theta)]$

$= [1 + \tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta] / [(1 + \tan^2\theta)]$

$= 2 (1 + \tan^2\theta) / [(1 + \tan^2\theta)]$

$= 2 \sec 2\theta$

$= 2 \cos 2\theta$

$= 2 / [a / b]$

$= 2b / a$

Question 10: The number of real solutions of $\tan^{-1} \sqrt{[x (x + 1)]} + \sin^{-1} [\sqrt{x2 + x + 1}] = \pi / 2$ is ____________.

Solution:

$\tan^{-1} \sqrt{[x (x + 1)]} + \sin^{-1} [\sqrt{x2 + x + 1}] = \pi / 2$

$\tan^{-1} \sqrt{[x (x + 1)]}$ is defined when $x (x + 1) \geq 0$ ..(i)

$\sin^{-1} [\sqrt{x2 + x + 1}]$ is defined when $0 \leq x (x + 1) + 1 \leq 1$ or $0 \leq x (x + 1) \leq 0$ ..(ii)

From (i) and (ii), $x (x + 1) = 0$ or $x = 0$ and -1.

Hence, the number of solution is 2.

Question 11: What is the value of $\sin (\cot^{-1} x)$?

Solution:

Let $\cot^{-1} x = \theta \Rightarrow x = \cot\theta$

Now $\text{cosec}\theta = \sqrt{[1 + \cot^2\theta]} = \sqrt{[1 + x^2]}$

Therefore, $\sin\theta = 1 / \text{cosec}\theta$

$= 1 / \sqrt{[1 + x^2]}$

$\Rightarrow \theta = \sin^{-1} \{1 / \sqrt{[1 + x^2]}\}$

Hence, $\sin (\cot^{-1} x) = \sin (\sin^{-1} \{1 / \sqrt{[1 + x^2]}\})$

$= 1 / \sqrt{[1 + x^2]}$

$= (1 + x^2)^{-1/2}$

Question 12: $\sec^2 (\tan^{-1} 2) + \text{cosec}^2 (\cot^{-1} 3) =$ _________.

Solution:

Let $(\tan^{-1} 2) = \alpha$

$\Rightarrow \tan \alpha = 2$ and $\cot^{-1} 3 = \beta$

$\Rightarrow \cot \beta = 3 \sec^2 (\tan^{-1} 2) + \text{cosec}^2 (\cot^{-1} 3)$

$= \sec^2 \alpha + \text{cosec}^2 \alpha$

$= 1 + \tan^2\alpha + 1 + \cot^2\alpha$

$= 2 + (2)^2 + (3)^2$

$= 15$

Question 13: A vertical pole consists of two parts, the lower part being one-third of the whole. At a point in the horizontal plane through the base of the pole and distance 20 meters from it, the upper part of the pole subtends an angle whose tangent is 1 / 2. The possible heights of the pole are _________.

Solution:

$[H / 3] \cot\alpha = d$ and $d = 150 \cot\phi = 60$m or

$[H / 3d] = \tan\alpha$ and

$[H / d] = \tan\beta$

$\tan (\beta - \alpha) = 1 / 2$

$= \{[H / d] - [H / 3d]\} / \{1 + [H^2 / 3d^2]\}$

$\Rightarrow 1 + [H^2 / 3d^2] = 4H / 3d$

$\Rightarrow H^2 - 4dH + 3d2 = 0$

$\Rightarrow H^2 - 80H + 3 * (400) = 0$

$\Rightarrow H = 20$ or 60m

Question 14: A tower of height b subtends an angle at a point O on the level of the foot of the tower and a distance a from the foot of the tower. If a pole mounted on the tower also subtends an equal angle at O, the height of the pole is ___________.

Solution:

$\tan\alpha = b / a$, $\tan^2\alpha = [2 (b / a)] / [1 - (b / a)^2] = [p + b] / a$

$\Rightarrow 2ba / [a^2 - b^2] = [p + b] / a$

$\Rightarrow [2ba^2 - a^2b + b^3] / [a^2 - b^2] = p$

$\Rightarrow p = [b * (a^2 + b^2)] / [a^2 - b^2]$

Question 15: A balloon is observed simultaneously from three points A, B and C on a straight road directly under it. The angular elevation at B is twice and at C is thrice that of A. If the distance between A and B is 200 metres and the distance between B and C is 100 metres, then the height of balloon is given by _________.

Solution:

$x = h \cot 3\alpha$ …..(i)

$(x + 100) = h \cot 2\alpha$ ……(ii)

$(x + 300) = h \cot\alpha$ ……(iii)

From (i) and (ii), $-100 = h (\cot 3\alpha - \cot 2\alpha)$,

From (ii) and (iii), $-200 = h (\cot 2\alpha - \cot\alpha)$,

$100 = h ([\sin\alpha] / [\sin 3\alpha * \sin 2\alpha])$ and $200 = h ([\sin\alpha] / [\sin 2\alpha * \sin\alpha])$ or

$\sin 3\alpha / \sin\alpha = 200 / 100$

$\Rightarrow \sin 3\alpha / \sin\alpha = 2$

$\Rightarrow 3 \sin\alpha - 4 \sin^3\alpha - 2 \sin\alpha = 0$

$\Rightarrow 4 \sin^3\alpha - \sin\alpha = 0$

$\Rightarrow \sin\alpha = 0$ or

$\sin 2\alpha = 1 / 4 = \sin^2 (\pi / 6)$

$\Rightarrow \alpha = \pi / 6$

Hence, $h = 200 * \sin [\pi / 3]$

$= 200* [\sqrt{3} / 2]$